

Optimal Spread with Heterogeneous Expectations in Foreign Exchange Market

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Abstract

Our subject is foreign exchange auction through market makers. We consider market maker's policy to choose bid and ask as an optimization problem. We show that if traders' expectations are more heterogeneous, the market maker widens bid and ask spread. He can take advantage of the heterogeneous expectations.

We derive an equation of expected transaction prices. This equation makes it possible that our representative market maker sets bid and ask so that expected profit per transaction is maximized. We apply our model to derive such bid and ask. Meanwhile the underlying model is about generation of order flow. Various authors, including Evans and Lyons (2002) documented significant effects of order flow in FX market. We fill what the existing literature lacks. We have mechanisms which generate stochastic arrivals of orders.

Two sources generate orders; FX dealers' revisions of expectations and retail transactions. Dealers with heterogeneous views as a whole absorb unbalanced arrivals of retail transactions. This determines path of transaction prices. FX market is not transparent. Dealers have to inquire market makers for quote. Only by that, they can obtain immediately executable quotes. Quotes may change during the search for another quotes. Using such an expected change, dealers determine reservation prices. As the expectations become more heterogeneous, so do reservation prices. They are dispersed over a wider range. Market makers can exploit it by wider spread.

1 Introduction

1.1 Motivation

Our subject is foreign exchange auction through market makers. We consider market maker's policy to choose bid and ask as an optimization problem. Our basic idea is that FX market maker can exploit heterogeneous expectations by increasing bid-ask spread. When the market is volatile, it may be that pieces of news are arriving sequentially. Also it may be in a process that traders converge to a consensus about a quantitative interpretation of a given news. In the latter case, the exploitation of heterogeneous interpretation likely to play larger role to set spread.

There is a brokered FX market, besides market makers'. The bid and ask may be quoted by different dealers. Those who submit limit orders are not obliged to have unwanted inventory. Still the spread widens when the market is volatile. This suggests the importance of viewing wider spread as a result of exploiting heterogeneous expectations, as well as compensation for taking unwanted position.

1.2 Equation of Expected Time Path

We derive an equation of expected time path of transaction price in FX market. As Lyons (2001) summaries, for example, macroeconomics models fail to explain price movements. However, dealers form expectations and assume positions. How do they do? Some of FX dealers give out suggestions on dealing business in their books. We translate their tips as follows. (a) They recognize heterogeneity of expectations. (b) Such heterogeneity takes a form of dispersion of positions. (c) The positions influence prices also when they are resolved, as well as when created. We incorporate the above (a) to (c) into our model. Dealers may share the same equation. However they have different parameter estimates. So they have heterogeneous expectations.

For a given dealer, expected price change while he searches for another quotation determines reservation price. The relevant reservation price is for ask, if he is bullish. To find the expected price change, we use the equation of expected time path. Value of this expected price change varies among dealers. Dispersion of the heterogeneous expectations determine that of values. More dispersed expectations imply that more of dealers accept a given value of bid and ask. Bid can be lower while ask can be higher, while market maker can expect the same expected number of transactions. Thus heterogeneous expectations give rise to wider bid and ask spread. Market maker take into account of competition between market makers. The competition also determines bid and ask spread.

Order flow, *i.e.*, signed transaction has significant effects on price in FX market. Evans and Lyons (2002) shows it. However, it does not suggest mechanism which generates order flow. Our paper fills what it lacks. Our model is about order flow generation. This makes it possible to derive the equation of expected time path

of transaction prices. Our model is in a context of continuous time auction. Our asynchronous transactions is in line with Garman (1976) and Amihud and Mendelson (1980). Comparing with them, we have explicit mechanisms which generate randomly arriving Garman's "statistical ensemble." Another difference is that we have multiple market makers in FX market. A given market maker can adjust his position right away by trading with other market makers. Our inventory control problem is simpler than theirs. We describe the transaction generating mechanism as a set of continuous time stochastic processes. Such a approach which has not been used looks useful to analyze microstructures. In many financial markets, as well as FX market, auctions are in continuous time.

Lyons (2001) discusses models about bid and ask spread in the existing literature. Three basic costs determine the spread: adverse selection costs, inventory costs, and order processing cost. Adverse selection in asset markets requires models to set equilibrium. However, how can we define equilibrium when quantitative interpretations about a given new does matter? We need intra-day equilibrium. The benchmark is missing. It requires such models to set benchmark about price also when market maker consider possible. By contrast with the approach to divide the spread into three components, our model has one component; reward for providing immediacy to those who are bullish and bearish.

1.3 Applicability

Not all of the market makers are active all the time. During low season such as before holidays or lunch time, some of the market makers virtually withdraw from the market making. Less competition for other market makers. The spread can become wider. Bollerslev and Domowitz (1993) reports that "the spread and market activity is at best vague and at worst conflicting." The market activity here means for market makers to revise own quotes and announce new pairs. This puzzle can be resolved. When the expectations are more heterogeneous and prices are more volatile, the spread widens. Also the quote revising frequency increases too. The spread and quote frequency increase together. Meanwhile, during low season, there is not much news giving rise to revising quotes. The spread increases while quote frequency decreases.

1.4 Environments of the Market

We consider a dealer who works for a FX market maker bank. We call him market maker. He is risk neutral. He is allowed to have intra-day open position up to a given size. It is not allowed overnight. His objective is to maximize daily profit. When inquired, he quotes two prices; buying and selling prices together. Both of them are executable immediately. The inquiring dealer does not specify in which side he is interested. So the market maker does not have control on his inventory in this sense. However, in the FX market, there are other market makers. He can

adjust his inventory to his choice if he acts as a customer of other market maker. Therefore with a little delay, he can control inventory. Adjusting competitiveness of his quotes is passive way of controlling his inventory.

His profit consists of two parts; first, reward for providing intermediacy and second, capital gain/loss of inventory. As intra-day volatility increases, the inventory has more risk. Meanwhile other traders would be more impatient. They are impatient in a sense that they do not want to miss the harbinger of a possible trend, or that they cannot wait betting on uncertain perspectives. With wider bid ask spread, the market maker still will find as many dealers as before who trade at his bid and ask.

The auction in FX market is continuous. The market maker has asynchronous and random arrivals of buyers and sellers. Here traded quantity is one unit per arrival. An expected value of traded quantity per unit time is called "intensity." The market maker faces the arrival intensities of buyer's and seller's. We call relationships between price and intensity "intensity curve." The intensity curves are continuous auction's counterparts of demand and supply curves. The difference is that a transaction does not necessarily take place even if price makes them actually matched. Buyers and sellers may arrive unevenly. The market maker holds unbalanced arrivals as inventory.

Buyers and sellers consist of two group of dealers. One group is non-market maker smaller banks. And the other is market maker banks. Banks, small and large collectively, absorb demand and supply of FX from economic fundamentals. These demand and supply emerge as banks' retail transactions. They are eventually passed on to the inter-bank market. Then dealers as a whole, absorb them. For a given amount of excess demand, necessary price change to absorb it depends on the dealers' expectations. Distribution of heterogeneous expectations does matter. All the dealers including market makers, recognize interactions between aggregated transaction arrivals and the distribution of heterogeneous expectations. They try to estimate these aggregate level parameters. Then they form expectations on the time path of the transaction price. By using the expected time path, dealers chooses their present action. The expected path also determines reservation price on bid or ask

Using expected flow of orders, we derive a differential equation of transaction price. Its solution makes it possible to model dealer's setting of reservation prices. Individual dealers thus determine reservation prices. The reservation prices are random variable. To express its distribution, the market maker uses tractable function. The distribution of reservation prices determines the intensity curves.

In a market makers' market, dealers search for favorable quote. The search is not exhaustive. The price, even if the best at that moment, does not necessarily takes all. Dealers randomly chooses two market makers. Then they inquire their quotes at the same time. They choose the best one. If none of them is acceptable, then

they continue the search. Such a search process with reservation price determines intensity curves. Competition with other market makers also determines intensity curves. Market maker estimates intensity curves. He chooses his quotes so that his expected profit per transaction is maximized.

In the following section we derive the equation of expected time path. We use uniform and triangular distributions to describe dispersions the reservation prices and other market makers' quotes. Other distribution functions are not tractable.

2 Model

2.1 Many Dealers

We have two types of dealers; type K and type L. Type K seeks capital gain. His objective is to maximize daily profit by taking position. Type L provides liquidity by acting as a market maker. They stand ready to trade any time at their quotations. Their intended profits consist of bid - ask spread. Taking open position is to provide liquidity, not to seek capital gains. We dichotomize the same person into type K and L. We assume the market maker acts only as type L of liquidity provider.

2.2 Generation of Transactions

Type K dealers trade in the market, based on their bullish or bearish expectations. Meanwhile they have retail transactions with their retail customers. There are two sources to generate transactions; expectation revisions and retail transactions.

Let's take representative type K dealer. Being bullish or bearish determines his optimal position. The key variable is the *first local extremum* of the expected time path of transaction price; denoted as FLE. We model transaction decision as follows. He revises this FLE from time to time. Expectation revision takes place when he moves between two states:

state 1: In state 1, He has FLE and assume open position based on it.

state 0: In state 0, He does not have FLE. He tries to keep closed position.

When he switches from state 0 to state 1, he picks FLE value. He tries to have open position based on FLE. He hits market maker's bid or ask. When he exits from state 1, he abandons his FLE. To close his position, he hits bid or ask. The expectation revising process thus generates transactions.

Dealers are all risk neutral. They are allowed to have one transaction unit of open position. If bullish or bearish, it must be the case that they have the maximum open positions. So when they have retail transactions, they have to counterbalance them in the inter-dealer market. Flow of retail transactions becomes flow in the inter-dealer market.

2.3 Stochastic Key Variables

Let T_0 and T_1 be sojourn time in state 0 and 1. They are random variables which follow exponential distribution with parameter θ_0 and θ_1 . Let I_j be an index function about the j th dealer's state of expectation. There are n_k of type K dealers. Let I_j for $j = 1, 2, \dots, n_k$ be random variables such that

$$I_j = \begin{cases} 0 & \text{if in state 0} \\ 1 & \text{if in state 1} \end{cases} \quad (1)$$

Let Z_j be the j th dealers position. It takes one of the three integer values; $Z_j = -1, 0, 1$. Negative value means short position. By regulations imposed by their banks, $|Z_j| \leq 1$. We define following random variables.

$$N_1 = \sum_j I_j \quad (2)$$

$$Z^- = \sum_j \frac{1}{2} (|Z_j| - Z_j) \quad (3)$$

Random variable Z^- as defined by (3) is the number of type K dealers who have short positions; $Z^- > 0$. It is also the size of the aggregated short positions. Let Z^+ be the number of those with long positions; $Z^+ > 0$.

Retail transactions are Poisson arrivals to a type K dealer. He tries to counter-balance them in the market. He does so because he does not want open position if he is in state 0. If in state 1, restriction on the open position must be already binding. Retail transactions are passed to the market right away. Sum of Poisson is also Poisson. Aggregated, retail transactions constitute Poisson arrivals. Let $R_d(t)$ and $R_s(t)$ be sums of the customers' buying and selling, accumulative over time interval $[0, t]$. We call $R(t)$ "excess demand" defined as follows;

$$R(t) = R_d(t) - R_s(t) \quad (4)$$

Let λ_d and λ_s be Poisson parameters for R_d and R_s . Then, $E[R_j(t)] = \lambda_j t + R_j(0)$ for $j = d, s$. Hence,

$$E[R(t)] = (\lambda_d - \lambda_s)t + R(0) \quad (5)$$

Let X_j be the j th dealer's FLE. We consider FLE's are distributed over a given finite interval. We consider that X_j is a random variable with uniform distribution on unit interval. And they are independently and identically distributed.

$$0 \leq X_j \leq 1 \quad (6)$$

2.4 Equations of Aggregate Position

If type K dealers trade only by different expectations, long and short positions must sum up to zero.

$$Z^- - Z^+ = 0 \quad (7)$$

When we introduce retail transactions, customers' demand for foreign exchange results in dealers' short positions. The following equations between the aggregated values hold.

$$Z^- + Z^+ = N_1 \quad (8)$$

$$R(t) = Z^-(t) - Z^+(t) \quad (9)$$

These variables take non-negative integer values. Although there are differences, depending on being even or odd, approximately we have

$$Z^- = \frac{1}{2}(N_1 + R) \quad (10)$$

2.5 Process of Expectation Revising

Switching between two states of expectation is continuous time Markov process. Index function I_j for $j = 1$ to n_k are *i.i.d.* variables. We use the derivation shown in Ross (1997). Then probability for the j th dealer to be in state 1 is given by

$$\lim_{t \rightarrow \infty} \Pr(I_j(t) = 1) = \frac{\theta_0}{\theta_0 + \theta_1} \quad (11)$$

Since I_j 's are *i.i.d.*, the expected number of dealers in State 1 is given by

$$\begin{aligned} E[N_1] &= \Pr(I_j(t) = 1)n_k \\ &= \frac{\theta_0 n_k}{\theta_0 + \theta_1} \quad \text{as } t \rightarrow \infty \end{aligned} \quad (12)$$

Suppose long enough time elapsed; counting time by second. Fix $E[N_1] = \frac{\theta_0 n_k}{\theta_0 + \theta_1}$. Let $n = \frac{\theta_0 n_k}{\theta_0 + \theta_1}$ to simplify notation. In the following, we treat $N_1 = n$.

We consider intensity to exit state 1. The intensity is expected number of those who exit per unit time. Let T_1 be sojourn time in state 1. Then T_1 follows exponential distribution with parameter θ_1 . Then intensity to exit from state 1 is given by θ_1 . The intensity is an increment of probability such that $T_1 < t$ conditional on $T_1 \geq t$. It is given by

$$\frac{f(t)}{1 - F(t)} \quad (13)$$

This is the same as what is called failure rate or hazard rate function in reliability theory. Since there are n of dealers in state 1, $\theta_1 n$ of them are expected to exit from state 1. As for entry into state 0, similar argument holds. There are $n_k - n$ dealers who are in state 0. Among them, $\theta_0(n_k - n)$ are expected to move from state 0 to state 1.

When dealers exit from state 1, they hit market maker's bid or ask to close positions. When entering, dealers pick up FLE value according to a given distribution function. They hit bid or ask depending on the picked FLE value. Retail transactions also give rise to hitting bid or ask. The intensity to hit bid due to retail transaction is given by λ_s and that for ask is λ_d .

2.6 Expected Flow of Orders

We arrange X_j 's from the smallest. Among X_j 's contained in $Z^-(t)$, we denote the maximum as X^- . We approximate movement of transaction price by that of $X^-(t)$. In other words, $X^-(t)$ is the of Z^- -th value of FLE from the smallest among those with $Z_j < 0$. Figure 1 at the end of paper provides graphical explanation .

Dealers hit bid or ask when they revise expectations. They do so in both cases of exiting from and entering into state 1. We consider the case of exit first. If $Z_j < 0$ then he hits ask. This is because, if he has short position until the revision, he has to hit ask in order to close his position. There are Z^- of dealers who have short positions. Hence, $\theta_1 Z^-$ of dealers are expected to hit ask. As for bid, Z^+ of dealers have long position. They will sell their inventory at a time of expectation revision. The expected number of those who hit bid is given by $\theta_1 Z^+$. Their intensities are as follows.

$$\text{when exiting from} \quad \text{hitting bid:} \quad \theta_1 Z^+(t) \quad (14)$$

$$\text{state1} \quad \text{hitting ask:} \quad \theta_1 Z^-(t) \quad (15)$$

Next we consider intensity to enter state 1. Using equation (12) and definition of n , intensity to enter state 1 is given by $\theta_0(n_k - n)$. When entering, they pick up values for FLE. These values may bullish or bearish. It depends on comparison the current price level. Then what is the current transaction price? We call a given length of moving average of transaction prices current transaction price. Let x be this current transaction price. Then we approximate x by X^- . This x sorts entrants into two groups as follows. For a given x , entrants' intensities are given by

$$\text{when entering into} \quad \text{hitting bid:} \quad \theta_0(n_k - n)x \quad (16)$$

$$\text{state1} \quad \text{hitting ask:} \quad \theta_0(n_k - n)(1 - x) \quad (17)$$

Then, using equation (12), net intensity to hit ask is given by

$$2\theta_1 n \left(\frac{Z^-(t)}{n} - x(t) \right) \quad (18)$$

2.7 Movement of Transaction Prices

Next we want to find expected change of $x(t)$ after time t . To simplify and conform usual notation, we reset time $t = 0$. By equation(5) and (10), for $t > 0$,

$$E[Z^-(t)] = \frac{1}{2}(n + R(0) + \lambda t) \quad (19)$$

$$= Z^-(0) + \frac{\lambda t}{2} \quad (20)$$

where $\lambda = \lambda_d - \lambda_s$. $Z^-(t)$ is expected to increase by $\frac{\lambda}{2}$ per moment by equation (5). Substitute this into the net intensity of (18). For $t > 0$, it becomes

$$2\theta_1 n \left(\frac{Z^-(0)}{n} - x(t) + \frac{\lambda t}{2n} \right) \quad (21)$$

For a given interval of Δx , the expected number of type K dealers whose x_j 's fall in the interval Δx is given by $\Delta x n$. The net intensity to hit ask is the expected number of dealers who joins Z^- . This net intensity to hit ask results in the number of K dealers who come to have short positions. Hence,

$$\Delta x n = 2\theta_1 n \left(\frac{Z^-(0)}{n} - x(t) + \frac{\lambda t}{2n} \right) \quad (22)$$

Then we have a differential equation;

$$x' = 2\theta_1 \left(\frac{Z^-(0)}{n} - x(t) + \frac{\lambda t}{2n} \right) \quad (23)$$

with an initial condition $x(0)$. The solution for equation (23) is given by, for $t > 0$,

$$x(t) = \left(x(0) - \frac{Z^-(0)}{n} \right) e^{-2\theta_1 t} + \frac{Z^-(0)}{n} + \frac{\lambda t}{2n} \quad (24)$$

2.8 Reservation Price for Ask

Type K dealers have reservation price with regard to market maker's bid or ask. We assume they go through the following process.

- a. Take representative dealer. He has bullish expectation and wants to have long position.
- b. He sets his reservation price equal to the expected price change for τ plus the current transaction price. Let a_k^* be his reservation price for ask.
- c. He contacts two market makers at the same time. Market makers quote a pair of bid and ask; b_j and a_j for $j = 1, 2$. Suppose $a_1 < a_2$. If $a_1 < a_k^*$, then he hits a_1 . If not, he searches for another pairs of quotations.
- d. It takes time τ until he obtains quotations from another pair of market makers.

As assumed in the above d, expected price change does matters to set reservation price. The expected change in transaction price is given by the following. Let $\Delta x = x(t + \tau) - x(t)$. Equation (24) gives expected time path.

$$\Delta x = \left(x(t) - \frac{Z^-(t)}{n} \right) (e^{-2\theta_1 \tau} - 1) + \frac{\lambda \tau}{2n} \quad (25)$$

$$\approx -2\theta_1 \tau \left(x(t) - \frac{Z^-(t)}{n} \right) + \frac{\lambda \tau}{2n} \quad (26)$$

What we obtained above is expected percentage of dealers who switch positions from long to short during τ .

Next we find change in transaction price in terms of currency. Let $W(t)$ be current transaction price denominated in currency terms. The current transaction price is meant to be moving average of the last several transaction prices. This variable is known to all the dealers. The j th dealer sets his reservation price at time t equal to $w(t)$ plus “expected change during next time τ ”. Transaction price $W(t)$ is random variable constructed by the following set of functions.

$$X = H(Y) \quad (27)$$

$$W = \rho_0 + \rho_1 Y \quad (28)$$

where $\rho_0, \rho_1 > 0$ and $0 \leq Y \leq 1$.

We introduce an inverse function of $F(y)$. Why do we introduce inverse function? In the above, we derived expected change of transaction price in terms of X_j . This random variable is uniformly distributed on unit interval. We want to derive expected change in terms of units of currency. We consider FLE has a distribution which is not uniform when we express them using units of currency. We want translate the result obtained using uniform distribution into expected change in units of currency. We will find $100x_j$ percentile value by inverse function. Let $H(x)$ be the inverse of $F(y)$;

$$Y = H(X) \quad (29)$$

where random variable X is uniformly distributed over unit interval. Function $H(x)$ takes x -quantile value of Y . Using equation(29),

$$\Delta y \equiv H(x(t+s)) - H(x(t)) \quad (30)$$

Random variable Y is standardized value of random variable W . Hence, applying definition,

$$\Delta w \equiv \rho_1 \Delta y \quad (31)$$

Let Δx_j^* be a change in x during time interval τ which the j th dealer’s expects. For a given Δx_j^* , Δw_j^* depends on $H(x)$ and ρ_1 . We characterize the heterogeneity of expectation by $F(y)$ and the value of ρ_1 . As the variance Y becomes larger, expectations are more heterogeneous among type K dealers. The same holds as ρ_1 becomes larger. For a given x_j , larger variance of Y and larger ρ_1 increase Δw_j^* . Type K dealers come to have more heterogeneous reservation prices.

For a given Δx_j^* , larger variance of Y and larger ρ_1 increase Δw_j^* . To show this effect we need analytical form of $H(x)$. $H(x)$ is the inverse of $F(y)$. Inverse of distribution function is not tractable in many cases. So we choose a linear combination of uniform distributions for $F(y)$. We show that, for a given Δx_j^* , if $F(y)$ changes so that it reflects more of heterogeneity, then Δw_j^* increases.

Here we take examples of $F_1(y)$ and $F_2(y)$.

$F_1(y)$ ’s density: $f_1(y) = 1$ And $h_1(y) = 1$ where h_1 is slope of $H_1(x)$

$F_2(y)$ ’s density: as shown below

$$f_2(y) = \begin{cases} \frac{2}{3} & \text{for } 0 \leq y \leq \frac{1}{4} \\ \frac{4}{3} & \text{for } \frac{1}{4} \leq y \leq \frac{3}{4} \\ \frac{2}{3} & \text{for } \frac{3}{4} \leq y \leq 1 \end{cases} \quad (32)$$

Hence, the slopes of the inverse are given by

$$h_2(y) = \begin{cases} \frac{3}{2} & \text{for } 0 \leq x \leq \frac{1}{6} \\ \frac{3}{4} & \text{for } \frac{1}{6} \leq x \leq \frac{5}{6} \\ \frac{3}{2} & \text{for } \frac{5}{6} \leq x \leq 1 \end{cases} \quad (33)$$

Since $H(x)$ is piece-wise continuous and linear, equation (30) is given by $H'(x(t))\Delta x$. Slope of $H_2(x)$ is less steep in the middle part; $h_2(x) < h_1(x)$, for $\frac{1}{6} \leq x \leq \frac{5}{6}$. Unless the transaction price stays in the unusual region, for a given Δx_j^* , Δy is smaller with $H_2(x)$. As for ρ_1 , equation (31) shows effect of larger ρ_1 .

2.9 Market Maker's Choice of Ask

Market makers and type K dealers all may agree on price determination mechanism of equation (24). They observe the same $w(t)$. However, their expectations become heterogeneous as follows. They may disagree each other about λ . Also they have different values for ρ_0 and ρ_1 of equation (28). The latter type of aspect becomes more influential after the news' arrival. Suppose $\lambda = 0$ in equation (24). Then suppose that news arrives. The first term of (24) tells that the transaction price is expected to move to the median. Estimate of this destination becomes more heterogeneous. FLE becomes more dispersed.

Next we like to derive market maker's optimal ask. Δx_j of (26) and hence Δw_j of (31) are random variables among type K dealers. Market maker tries to approximate dispersion of Δw_j . Let $U = \Delta w_j$ to simplify notation. Suppose that he assigns a simple form of distribution $G(u)$ to Δw . He considers that $G(u)$ has support interval $[0, u_0]$, where $u_0 > 0$.

Type K dealer contacts two market makers at the same time. Ask price consists of two parts; $w(t)$ and u . Our market maker tries to figure out what will be his competitor's value of u . He assigns distribution function $J(u)$ to it. He considers that $J(u)$ has the same support as $G(u)$. Probability that his price is lower than dealer's reservation price is $1 - G(u)$. Probability that his ask is lower than the competitor's is $1 - J(u)$. Thus, ask's intensity curve is given by $(1 - G(u))(1 - J(u))$. Hence, the expected profit π_a by setting ask price at $w(t) + u$ is given by

$$\pi_a(u) = u(1 - G(u))(1 - J(u)) \quad (34)$$

where we use $w(t)$ to evaluate cost.

He chooses the value of u which maximizes equation (34). Let u^* be the optimal value of u . He sets his ask at $w(t) + u^*$, when the restriction on open position is

not binding. The heterogeneity of expectation determines u^* through the function $G(u)$. So do competitions with other market makers through the term of $J(u)$.

We measure the degree of heterogeneity by the variance of Y of (28) and by ρ_1 of (28). Their larger values imply type K dealers' expectations are more heterogeneous. The larger variance Y increases probability of large Δy . Larger ρ_1 stretches Δy proportionately. The market maker recognizes these causalities. When the expectations are heterogeneous, the market maker increases the value of u_1 ; making the support of $G(u)$ and $J(u)$ wider. As type K dealers' expectations become more heterogeneous, u^* increases.

He chooses different value if the restriction on the open position is binding. Let Z_m denote market maker's position. If $Z_m = -2$, then he sets his ask at $w(t) + u_0$. According to his calculation, transaction will not take place at ask. If $Z_m = 2$, then he has to increase probability of selling while lowering that of buying. He chooses $u = 0$ while setting probability of bid to be zero.

Similar argument holds for bid rate too. Hence, the bid ask spread becomes wider, as the expectation among dealers become more heterogeneous. The spread becomes wider, because market makers can exploit heterogeneous expectations.

2.10 Model Applications with Distribution Functions

Suppose that $G(u)$ is uniform distribution and that $J(u)$ is symmetric triangular distribution. Then $\pi_a(u)$ of equation (34) is maximized at $u^* = \frac{u_0}{\sqrt{6}}$. As u_0 increases, so does u^* .

We can show the effect of competition by modifying equation (34). If K dealer contacts m market makers at the same time, then Equation (34) is replaced by becomes smaller as the number of competitor increases.

$$u^* = \frac{u_0}{\sqrt{2 + 4(m - 1)}}. \quad (35)$$

The market sometimes becomes less competitive. One of the examples is a day before long holidays. The spread widens. Some of the market makers are not really active. In this case, we can consider that they choose value of u so that they do not have transaction. The shape of $J(u)$ changed. Its density is triangular with right angle at u_0 . The actively trading market maker's optimal value for equation (34) is given by $u^* = \frac{2u_0}{\sqrt{6}}$. The spread widens.

3 Conclusions

In our paper, heterogeneity of expectations means the dispersion of FLE; the first local extremum on the expected time path of the transaction price. We showed that, when expectations are more heterogeneous, market maker's optimal spread widens. The spread widens because the market maker still expects as many transactions as before. He is taking advantage of heterogeneous expectations. Applying our model, we can clarify reasons behind the seeming paradox suggested by Bollerslev and Domowitz (1993). The changing degrees of expectation's heterogeneity and of competitions could be attributable to their observations.

What are the advantages of trading foreign exchange with market makers as opposed to participating in the brokered auction? One of them can be "lack of market impact." When you trade with market makers, you do not face it. They are ready to trade greater than usual amount at own quotes. You can trade with them at the same time. Price changes after you trade. Dealers may choose trading with market makers. So variability of trading quantity is one of the characteristics of market makers' market. In this paper, however, we fixed quantity at one unit. We have not yet modeled the advantageous characteristics of market makers' market. To do so, we need treat retail transactions as compound Poisson process rather than Poisson. In case of compound Poisson, the associated quantity is random variable.

There is another issue we have not modeled. It is the market maker's position control when we assume he seeks capital gain while providing liquidity service. This issue and introducing compound process complicate model. However, the extending model will help understand seemingly plain random price formation in foreign exchange market.

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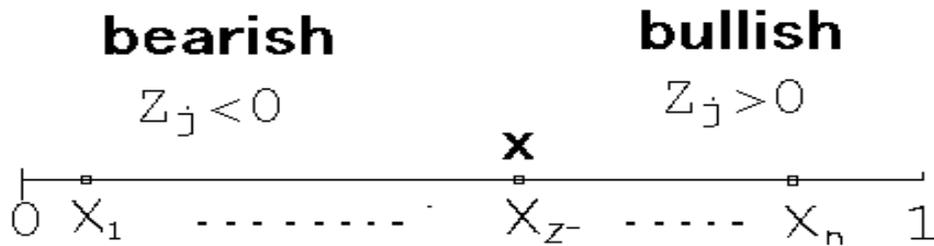


Figure 1: **Distribution of x_j 's and Open Positions** The current transaction price is denoted by x . FLE values are renumbered from the smallest. If $x_j < x$, then the j th dealer is bearish and has short position. Because of restriction on open position, $|Z_j| \leq 1$. the number of dealers who have short position coincides with the aggregate short position. If a given dealer has short position, he will hit ask when he abandons his FLE. He abandons it when he exits from state 1.